In this example design, where the diplexer is required for telemetry, the crossover region is relatively unimportant and the responses cross at about 6 dB, with accompanying poor VSWR. Where maximum usage of bandwidth is essential, as in communication channels, more filter sections are necessary, with crossover designed at 2–3 dB. Under such situations the annulling network is adequate in providing acceptable VSWR figures.

The principal advantage of these filters lies in the large range of volume/loss tradeoff obtainable, permitting losses comparable with conventional waveguide, when larger dimensions may be tolerated. In this particular application, however, the evanescent-mode filter offers no loss advantage over a comparable volume TEM filter, but does have a substantial weight advantage, a prime consideration in satellites. This advantage derives from its use of lightweight material and from the simplicity of the annulling network, part of which (the inductive element) is intrinsic to the structure.

V. Spurious Passbands

These occur above the cutoff frequency of the waveguide. In the above diplexer the first parasitic passband occurs at around 7.3 GHz, which is more than $3f_0$. If necessary, this figure may be improved through the use of smaller size waveguides to 4–$5f_0$ at the expense of loss. Indeed simple broadband (~30 percent bandwidth) evanescent-mode filters with 3 or 4 cavities, built in guides way below cutoff have been used effectively to replace the much more expensive waffle-iron filters for spurious suppression in low power applications [8].

VI. Conclusion

A design procedure for diplexers with no guard band, in waveguides below cutoff, has been described. Based on this a diplexer has been constructed and tested, yielding results that are in agreement with computed values. The design is valid for bandwidths of up to a few percent and, as it stands, is specifically for coaxial termination at the common port; however, it can be extended to include other terminations at this port. The integral construction of the diplexer, besides its simplicity, results in additional saving in weight in an already lightweight medium. Further size and weight reductions may be obtained through the use of inductive loading [9], with little sacrifice in loss. The small size, light weight, low-loss, and good temperature stability of these units should make them highly attractive in aerospace applications.

Acknowledgment

The author wishes to thank G. Craven for reading the manuscript and for useful comments, and Standard Telecommunication Laboratories Ltd. for permission to publish the paper.

References

[1] R. J. Wenzel, "Application of exact synthesis methods to multichannel filter design," *IEEE Trans. Microwave Theory Tech.* (*1964 Symposium Issue*), vol. MTT-13, pp. 5–15, Jan. 1965.

[2] R. G. Veltrop and R. B. Wilds, "Modified tables for the design of optimum diplexers," *Microwave J.*, vol. 7, no. 6, pp. 76–80, June 1964.

[3] G. L. Matthaei and E. G. Cristal, "Theory and design of diplexers and multiplexers," in *Advances in Microwaves*, vol. 2. New York: Academic Press, 1967, pp. 237–326.

[4] G. Craven and C. K. Mok, "Lightweight microwave components for communication satellites" presented at the Symp. Long Term Prospects for Communication Satellites, Genoa, Italy, June 1971.

[5] G. Craven, "Waveguide below cut-off: A new type of microwave integrated circuit," *Microwave J.*, vol. 13, pp. 51–58, Aug. 1970.

[6] G. A. Hockham, "Flush mounted aeronautical antenna," presented at the IEE Conf. Aerospace Antennas; also in *Conf. Pub. No. 77*, June 1971, pp. 49–53.

[7] G. F. Craven and C. K. Mok, "The design of evanescent mode waveguide bandpass filters for a prescribed insertion loss characteristic," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 295–308, Mar. 1971.

[8] G. Craven, "Slimguide microwave components," *Elec. Commun.*, vol. 47, no. 4, Nov. 1972.

[9] C. K. Mok, D. W. Stopp, and G. Craven, "Susceptance loaded evanescent mode waveguide filters," *Proc. Inst. Elec. Eng.*, vol. 119, no. 4, pp. 416–420, Apr. 1972.

Short Papers

Equivalent Circuit of Reentrant Cavity

KATSUAKI UENAKADA

***Abstract*—In order to develop a design formula for a reentrant cavity, as it changes from the very flat to very long form, the admittance of the cavity at the gap is derived. For the resonant frequency of an air-filled cavity, the theoretical value by the formula derived here agrees well with the experimental value.**

Manuscript received October 12, 1971; revised July 28, 1972.
The author is with the Technical Research Laboratories of Nippon Hoso Kyokai, Tokyo, Japan.

I. Introduction

Reentrant cavities have advantages of simple mechanical construction and wide tuning range, for which they have been used effectively in klystrons. With recent developments in solid-state devices, such as Gunn diodes and varactors, there has been a widespread use of these cavities in the form shown in Fig. 1(a).

In Fig. 1(a), the cylindrical coordinates ρ, ϕ, and Z are used as shown. The radial-line region containing the active solid-state element delineated by $0 \leqq \rho \leqq r_1$, $0 \leqq Z \leqq d$ is designated herein as region A, and the coaxial cavity region bound by $r_2 \geqq \rho \geqq r_1$, $0 \leqq Z \leqq l$, as region B. The admittance Y_a of a cavity having the active solid-state element can be calculated from an analysis of a radial line [5].

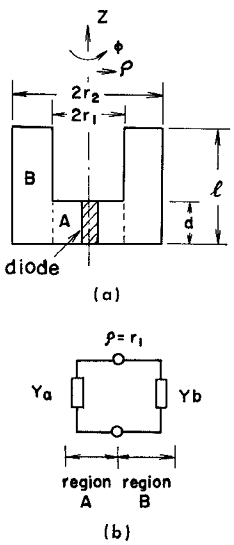


Fig. 1. (a) Cross-sectional view of cavity. (b) Equivalent network.

Thus the remaining problem is to calculate the admittance Y_b of the coaxial cavity region B looking outward from the reference cylinder at $\rho=r_1$. An expression for the admittance Y_b of region B has been previously derived by Fujisawa [1].

The results of this theory are likely to be in error if the shape of the cavity is either too long or too flat. Rivier [2], on the other hand, derived the equivalent lumped elements by use of the radial-line mode. The results of this theory are likely to be in error if the shape of the cavity is too long.

This paper provides a theoretical solution of the admittance presented at the gap ($\rho=r_1$, $0\leqq Z\leqq d$) of the cavity.

II. Theoretical Study of the Reentrant Cavity

The calculation of the admittance at the reference surface (which is the boundary between the two regions) can be reduced to a Kirchhoff problem of determining the field excited by an equivalent magnetic-current distribution M flowing on a perfectly conducting surface (Fig. 2).

This problem is equivalent to calculation by the method of Felsen and Marcuvitz [3] of the excitation of spherical waves by a slot in a rectangular waveguide. In this cavity, the mode has angular symmetry and the angular component of the electric field E_ϕ is zero.

Since the electric field on the surface of S_a ($\rho=r_1$, $0\leqq Z\leqq d$) can be expressed as $\boldsymbol{E}=\boldsymbol{a}_\rho E_\rho+\boldsymbol{a}_z E_z$, the equivalent magnetic current $\boldsymbol{M}(=\boldsymbol{E}\times\boldsymbol{a}_\rho)$ is given by

$$\boldsymbol{M}(\equiv\boldsymbol{a}_\phi M)=\boldsymbol{a}_\phi E_z \tag{1}$$

where $\boldsymbol{a}_\rho$, $\boldsymbol{a}_\phi$, and $\boldsymbol{a}_z$ are cylindrical-coordinate unit vectors. The field equation in region B is given by

$$\frac{\partial^2 H\phi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial H\phi}{\partial\rho}+\frac{\partial^2 H\phi}{\partial Z^2}+\left(k_0^2-\frac{1}{\rho^2}\right)H\phi=j\omega\epsilon M. \tag{2}$$

The boundary condition on all conducting surface is

$$\frac{\partial}{\partial n}(\rho H\phi)=0 \tag{3}$$

where $\partial/\partial n$ denotes differentiation in the direction normal to the conducting surface, and $k_0=\omega\sqrt{\mu\epsilon}$ is the wavenumber in free space.

The Green's function $G(\rho, Z/\rho', Z')$ corresponding to (2) is defined as follows:

$$\frac{\partial^2 G}{\partial\rho^2}+\frac{1}{\rho}\cdot\frac{\partial G}{\partial\rho}+\frac{\partial^2 G}{\partial Z^2}+\left(k_0^2-\frac{1}{\rho^2}\right)G=\frac{j\omega\epsilon\delta(\rho-\rho')\delta(Z-Z')}{2\pi\rho} \tag{4}$$

$$\int_{-\infty}^{\infty}\delta(\rho-\rho')\,d\rho=1\qquad\int_{-\infty}^{\infty}\delta(Z-Z')\,dZ=1.$$

If we now solve the Green's function considering the boundary condition, we get [4]

$$\begin{aligned}G(\rho, Z/\rho', Z')&=j\sum_{i=0}^{\infty}D_i(\rho,\rho')\cdot\cosh\Gamma_i(l-Z)\cdot\cosh\Gamma_i Z',\quad Z\geqq Z'\\&=j\sum_{i=0}^{\infty}D_i(\rho,\rho')\cdot\cosh\Gamma_i(l-Z')\cdot\cosh\Gamma_i Z,\quad Z\leqq Z'\end{aligned} \tag{5}$$

where

$$\begin{aligned}D_i(\rho,\rho')&=\frac{1}{2\pi\rho\rho'\eta\ln\dfrac{r_2}{r_1}\cdot\sin k_0 l},\qquad i=0\\&=-\frac{k_0 x_i^2 Q_i\left(\dfrac{x_i}{r_1}\rho\right)Q_i\left(\dfrac{x_i}{r_1}\rho'\right)}{2\pi r_1^2\eta\Gamma_i\cdot\sinh\Gamma_i l\cdot\displaystyle\int_{x_i}^{(r_2/r_1)x_i}xQ_i^2(x)\,dx},\\&\qquad i=1,2,\cdots,\infty\end{aligned}$$

$\eta=\sqrt{\dfrac{\mu}{\epsilon}}$ free space impedance,

$Q_i(y)$ $\{N_0(x_i)J_1(y)-J_0(x_i)N_1(y)\}/J_0(x_i)$,

$J_P(y)$ Bessel function of order P

$N_P(y)$ Neumann function of order P.

The quantities $x_i(i=1,2,\cdots,\infty)$ are roots which satisfy

$$J_0(x_i)\cdot N_0\left(\frac{r_2}{r_1}x_i\right)-N_0(x_i)\cdot J_0\left(\frac{r_2}{r_1}x_i\right)=0,\quad i=1,2,\cdots,\infty. \tag{6}$$

Γ_i is obtained by the relation

$$\begin{aligned}\Gamma_i&=jk_0,\qquad i=0\\&=\sqrt{\left(\frac{x_i}{r_1}\right)^2-k_0^2},\qquad i=1,2,\cdots,\infty.\end{aligned} \tag{7}$$

The magnetic field in the region B excited by the equivalent magnetic current M can be calculated using the Green's function given by (5), and is as follows:

$$H_\phi=2\pi r_1\int_0^d G(\rho, z/r_1, z')M(z')\,dz'. \tag{8}$$

The admittance Y_b of the region B at the reference surface, located on the S_a surface at $\rho=r_1$, $0\leqq z\leqq d$, can be expressed in the following variational form:

$$Y_b=-\frac{4\pi^2 r_1^2\displaystyle\int_0^d\int_0^d M(z)G(r_1, z/r_1, z')M(z')\,dz\,dz'}{\left\{\displaystyle\int_0^d M(z)\,dz\right\}^2}. \tag{9}$$

For the true value of $M(z)$ the absolute value of Y_b is minimum and stationary. That is to say, an approximation to first order in $M(z)$ gives an approximation to second order in Y_b. Let

$$M_{(N)}(z)=\sum_{m=0}^{N}a_m\cos\frac{m\pi z}{d},\qquad N\geq 0. \tag{10}$$

We denote by $Y_{b(N)}$ the value obtained by substituting $M_{(N)}(z)$ for $M(z)$ in (9).

By the Rayleigh–Ritz method [5] the amplitude coefficients a_m are chosen so as to yield a minimum value of $|Y_{b(N)}|$ for every m.

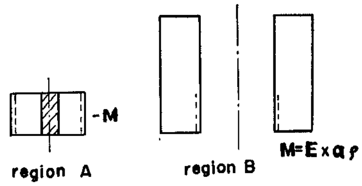


Fig. 2. Equivalent geometries.

The minimizing coefficients satisfy the following set of N equations

$$\frac{\partial Y_{b(N)}}{\partial a_m} = 0, \qquad m = 1, 2, \cdots, N \tag{11}$$

for

$$a_0 = 1.$$

Thus the admittance $Y_{b(N)}$ is obtained by

$$Y_{b(N)} = -j\left(\frac{2\pi r_1}{d}\right)^2 \left\{\sum_{m=0}^{N} A_{mm}a_m^2 + \sum_{\substack{m=0}}^{N}\sum_{\substack{\nu=0\\ m\neq\nu}}^{N} A_{m\nu}a_m a_\nu\right\} \tag{12}$$

where

$$A_{m\nu} = \sum_{i=0}^{\infty} D_i(r_1, r_1)\left\{\frac{\Gamma_i d^3 \cdot \sinh \Gamma_i l}{\epsilon_m(\Gamma_i^2 d^2 + m^2\pi^2)} - \frac{\Gamma_i^2 d^4 \cdot \sinh \Gamma_i d \cdot \sinh \Gamma_i(l-d)}{(\Gamma_i^2 d^2 + m^2\pi^2)^2}\right\}, \qquad \nu = m$$

$$= \sum_{i=0}^{\infty} \frac{-D_i(r_1, r_1)\Gamma_i^2 d^4 \cdot \cos m\pi \cdot \cos \nu\pi \cdot \sinh \Gamma_i d \cdot \sinh \Gamma_i(l-d)}{(\Gamma_i^2 d^2 + m^2\pi^2)(\Gamma_i^2 d^2 + \nu^2\pi^2)}, \qquad \nu \neq m$$

$$\epsilon_m = 1 \quad m = 0$$
$$= 2 \quad m \neq 0.$$

Using (11), the coefficients $(a_1, a_2, \cdots, a_N)$ are determined by the following set of N equations

$$\sum_{s=1}^{N} A_{ps}a_s = -A_{0p}, \qquad p = 1, 2, \cdots, N. \tag{13}$$

An approximate expression $\tilde{Y}_b$ for the admittance $Y_{b(N)}$ can be obtained from (12), if we assume that the magnetic current is uniform for the Z direction and let $x_i \gg 1$ $(i = 1, 2, \cdots, \infty)$ and $k_0(r_2 - r_1) \ll \pi$. Thus we obtain

$$\tilde{Y}_b = -j\frac{1}{Z_0 k_0 d}\left\{1 - \frac{\sin k_0(l-d) \cdot \sin k_0 d}{k_0 d \cdot \sin k_0 l}\right\} + j\frac{4r_1 g}{\eta d}\left\{0.65 + \frac{1}{1-g^2} - \frac{g\xi}{k_0 d} \cdot \frac{5.32 - 3.4g^2}{4 - 5g^2}\right\} \tag{14}$$

where

$$Z_0 = \frac{\eta}{2\pi}\ln\frac{r_2}{r_1}$$
$$= k_0/T$$
$$T = \frac{\pi}{r_2 - r_1}$$
$$\xi = \tfrac{1}{2}\{1 - e^{-2T(l-d)} - e^{-2Td} + e^{-2Tl}\}.$$

The admittance $Y_{a(N)}$ of the region A for the magnetic current of (10) without the solid-state element is obtained by

$$Y_{a(N)} = j\frac{2\pi r_1}{\eta d}\left\{\frac{J_1(k_0 r_1)}{J_0(k_0 r_1)} + \sum_{m=1}^{N}\frac{k_0 a_m^2 I_1(K_m r_1)}{2K_m I_0(K_m r_1)}\right\} \tag{15}$$

where $I_p(K_m r_1)$ represents the modified Bessel function of first kind of order p.

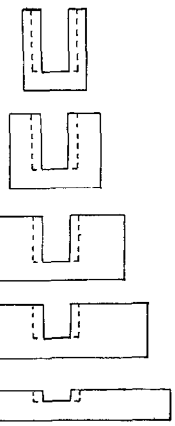
$$K_m = \sqrt{\left(\frac{m\pi}{d}\right)^2 - k_0^2}.$$

An approximate expression $\tilde{Y}_a$ for the admittance $Y_{a(N)}$ can be obtained from (15), if we assume that the magnetic current is uniform for the Z-direction and let $k_0 r_1 \ll 1$. Thus we obtain

$$\tilde{Y}_a = j\frac{\pi k_0 r_1^2}{\eta d}. \tag{16}$$

III. Experimental Measurements of Resonant Frequencies

For the purpose of experimental verification of our formulas, an experimental cavity was built with just an air gap. The constructions and the measured resonant frequencies are shown in Fig. 3. In Fig. 4,



Dimensions (mm)				(GHz)	
r_2	l	r_1	d	fm	l/r_2
13.80	34.83	10.00	7.97	2.4146	2.5239
		6.00	7.99	2.3270	
20.00	32.02	10.00	8.00	2.2494	1.6010
		6.00	8.00	2.3014	
30.00	28.00	10.00	7.99	2.0645	0.9333
		6.00	8.01	2.2321	
40.00	23.01	10.00	7.99	2.0071	0.5752
		6.00	8.01	2.1907	
49.74	13.01	10.00	8.01	2.1625	0.2615
		6.00	8.02	2.2152	

fm = measured

Fig. 3. Construction and measured resonant frequency of experimental cavities.

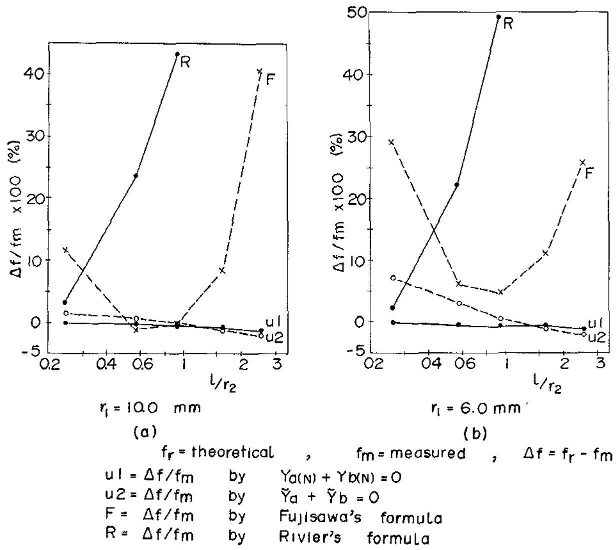


Fig. 4. Percentage errors of the calculated resonant frequencies of the cavities shown in Fig. 3.

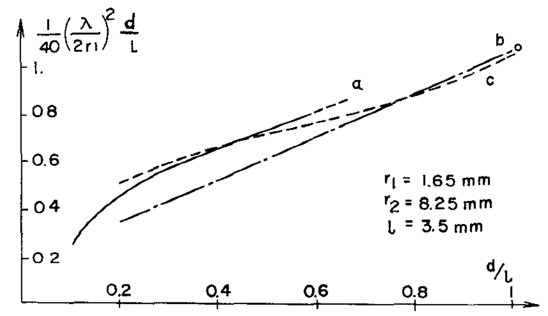


Fig. 5. Comparison between Rivier's experimental a and theoretical b curves and theoretical $(\tilde{Y}_a + \tilde{Y}_b = 0)$ curve c.

the calculated resonant frequencies obtained by Fujisawa's formula [1], Rivier's formula [2], and our two formulas $(Y_{a(N)} + Y_{b(N)} = 0$, $\tilde{Y}_a + \tilde{Y}_b = 0)$ are compared with results of the experiments on the ten cavities. The theoretical values derived by our formulas are in good agreement with the experimental values.

In Fig. 5 comparisons are made between two theoretical values [our approximate formula $(\tilde{Y}_a + \tilde{Y}_b = 0)$ and Rivier's formula] and

Rivier's experimental values [2]. By comparing the curves, our formulas are found to agree well with Rivier's experimental results.

ACKNOWLEDGMENT

The author wishes to thank Dr. K. Suetake of Tokyo Institute of Technology and Dr. Y. Konishi of Japan Broadcasting Corporation.

REFERENCES

[1] K. Fujisawa, "General treatment of klystron resonant cavities," *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, pp. 344–358, Oct. 1958.
[2] E. Rivier and M. V. Lapisardi, "Lumped parameters of a reentering cylindrical cavity," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 309–314, Mar. 1971.
[3] L. B. Felsen and N. Marcuvitz, "Slot coupling of rectangular and spherical wave guides," *J. Appl. Phys.*, vol. 17, p. 1047, Dec. 1946.
[4] K. Uenakada, "An *LCR* equivalent circuit of reentrant cavity and its application for parametric amplifier," *NHK Tech. J.*, vol. 22, no. 4, p. 32, 1970.
[5] W. J. Getsinger, "The packaged and mounted diode as a microwave circuit," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-14, pp. 58–69, Feb. 1966.
[6] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960, p. 232.

Resistivity of Thin Metal Films

H. K. CHAURASIA AND W. A. G. VOSS

***Abstract*—It is shown that the sheet resistance, and hence the resistivity, of very thin metal films (<100 Å) can be determined conveniently and accurately by microwave measurements. Accuracy is limited by VSWR measurement, film-holder design, and short-circuit quality. DC and microwave resistivity measurements are given for gold films on cleaved mica.**

I. INTRODUCTION

A waveguide impedance method due to Slater [1] for measuring the conductivity of metal films has been used by Clark [2]. This method is simpler than the field approach, which has been used for metal and semiconductor films on thick substrates [3]–[6], and provides a useful technique for determining the conductivity of ultrathin (<100 Å) metal films.

As the film thickness becomes comparable to the substrate surface irregularities, its macroconductivity is as much a function of the film structure as the microtopography of the substrate. Cleaved mica faces, the smoothest available, show topographic irregularities of the order of the lattice parameters [7]. These are at least seven orders of magnitude smaller than a wavelength at 10 GHz, and as such will not affect the wave.

A film of thickness $l \ll \delta$, the skin depth, and bulk conductivity σ, placed across a rectangular waveguide operating in the TE_{10} mode, create an admittance $\sigma l + Y_t$, where Y_t is the admittance of the waveguide termination at the film and is zero when a perfect short is placed at $z=0$ ($\lambda_g/4$ behind the film in Fig. 1). The conductance σl then corresponds to the conventional definition of the dc sheet resistance, i.e., $R_s = 1/\sigma l$ $\Omega/\square$. The microwave value $R_s(\mu)$ can be measured as rZ_0 when $R_s(\mu) \geq Z_0$, or Z_0/r when $R_s(\mu) \leq Z_0$, where r is the VSWR and Z_0 is the wave impedance at the operating frequency. Replacing the film by a short circuit at $z = -\lambda_g/4$, any reactive part resulting from a significant discontinuity due to the substrate and its mounting in the waveguide will be indicated by a minima shift other than zero ($R_s(\mu) = Z_0/r$) or $\lambda_g/4$ ($R_s(\mu) = rZ_0$).

It can be shown that a thin, lossless dielectric sheet of thickness d and relative permittivity ϵ will cause a VSWR given by [8]

$$r_\epsilon = 1 + \frac{2\pi d}{\lambda}\left[\sqrt{\{\epsilon - (\lambda/\lambda_c)^2\}/\{1 - (\lambda/\lambda_c)^2\}} - 1\right] \qquad (1)$$

where λ and λ_c are the free-space and cutoff wavelengths, respectively.

The accuracy with which the resistivity $\rho_\mu = lR_s(\mu)$ can be determined thus depends on 1) making $l \ll \delta$; 2) $Y_t = 0$, demanding an ideal short circuit and lossless waveguide; 3) the accuracy of measuring r, particularly when $R_s(\mu)$ is much different from Z_0; and 4) the effects of the discontinuity caused by the substrate and film holder.

Manuscript received June 14, 1971; revised June 14, 1972.
The authors are with the Department of Electrical Engineering, University of Alberta, Edmonton, Alta., Canada.

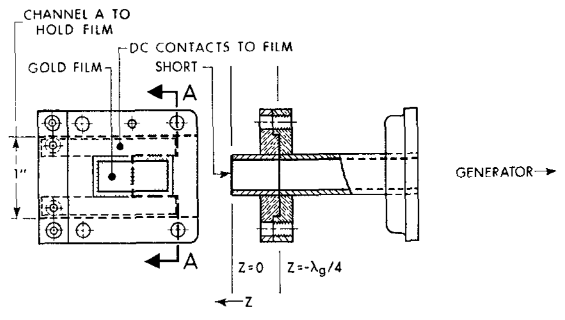


Fig. 1.

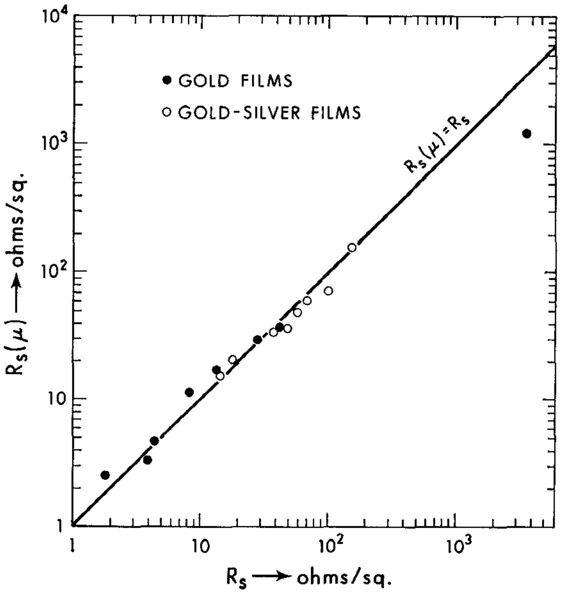


Fig. 2.

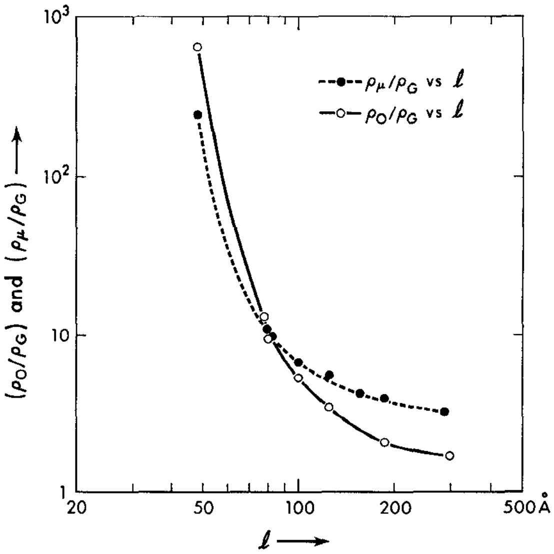


Fig. 3.